Пространственно-распределенные эволюционные игры с нелокальной информацией: среднее поле

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Механизмы эволюционного возникновения кооперативного поведения в популяциях рациональных агентов.

“Не обманешь, не продашь”

Tragedy of the commons

Конкуренция за общий ресурс приводит к его истощению
(Lloyd, 1833; Hardin, 1968)

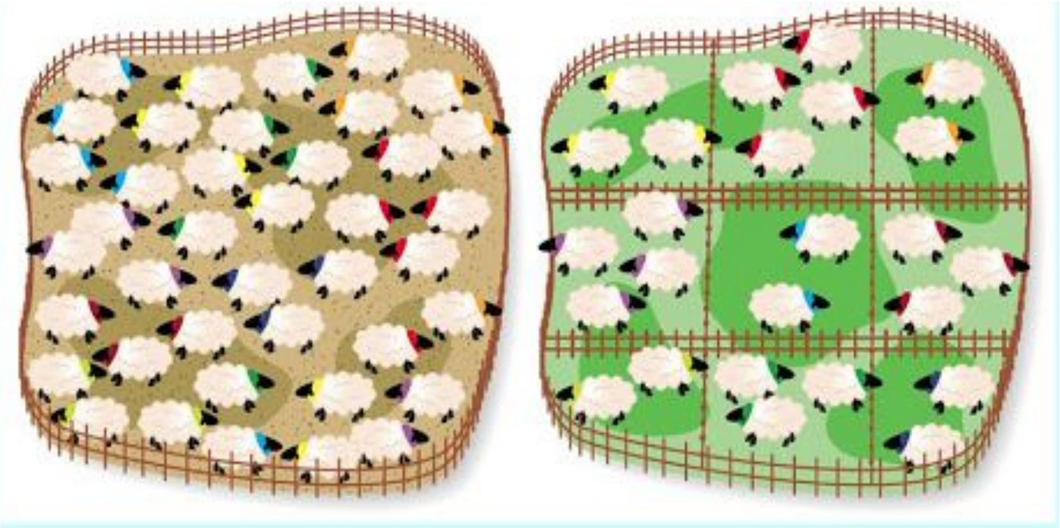
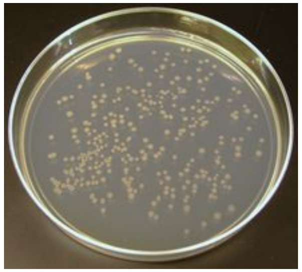


Image: Net Economy, D.S. Wilson)



Prisoner's dilemma

Two players. Each has two possible strategies: *cooperate* (**C**) or *defect* (**D**).

		Агент	
		$\mathcal{D}$	$\mathcal{C}$
Сосед	$\mathcal{D}$	0	0
	$\mathcal{C}$	b	1

In general, there are four possible combinations.

Here: a single free parameter, *b*.

Prisoner's dilemma

Two players. Each has two possible strategies: *cooperate* (**C**) or *defect* (**D**).

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Сосед	$\mathcal{D}$	0	0
	$\mathcal{C}$	b	1

A single game:

$b > 1$ D wins

Repeated game:

Imitation, tit-for-tat etc

Spatial games

Traditional game theory: two smart players, develop the optimal strategy in response to past encounters.

Spatial games: macroscopic number of simple players, arranged in some spatial structure, play repeatedly.
Look for emergent patterns *of strategies*.

Simple players: change strategies in a simple, predefined way. *E.g.*, imitate your neighbor, go with the winner etc.

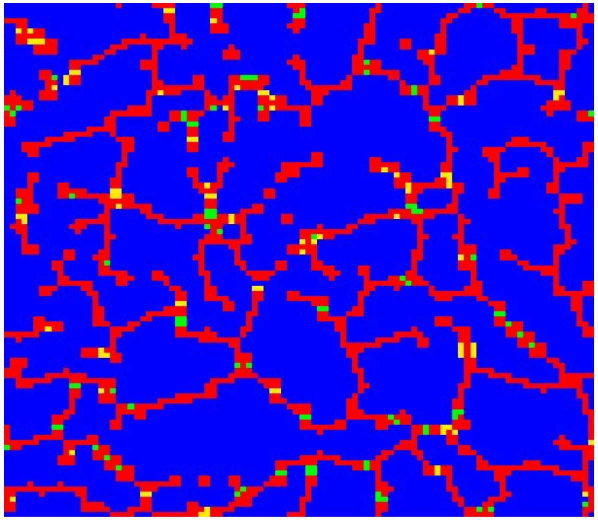
Evolution of cooperation:

Axelrod, 1984
Nowak and May, 1992
Hauert and Szabo, 2005
Szolnoki and Perc, 2013
Helbing, 2013

The game of Nowak and May

Short-memory, maximally opportunistic players (*Nowak and May, 1992*):

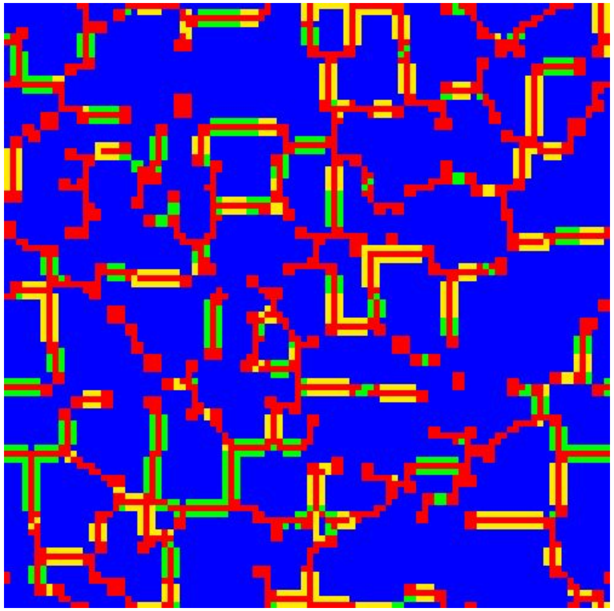
- $L \times L$ players arranged in a square grid.
- The game is played in discrete time steps
- In each round, each agent plays 9 games (8 nearest neighbors + itself)
- In the next round, a player adopts the strategy of the opponent with the largest payoff in the last round.



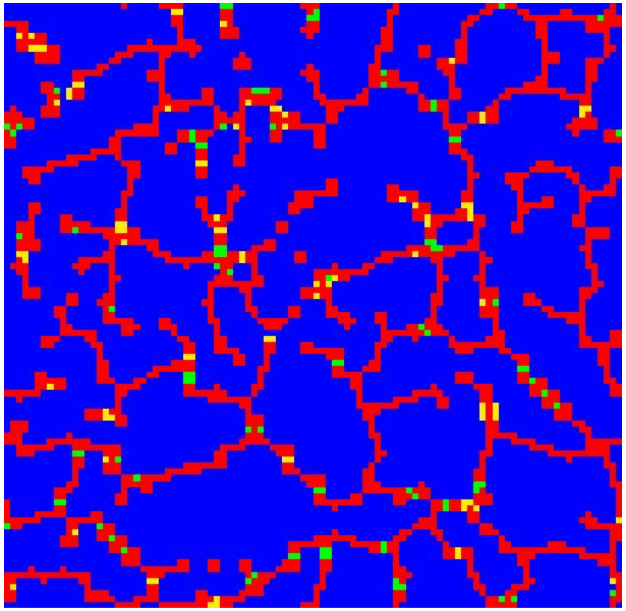
- Given the initial conditions, the dynamics is deterministic, and is **strongly dependent on *b***

Game field configurations (long time steady state)

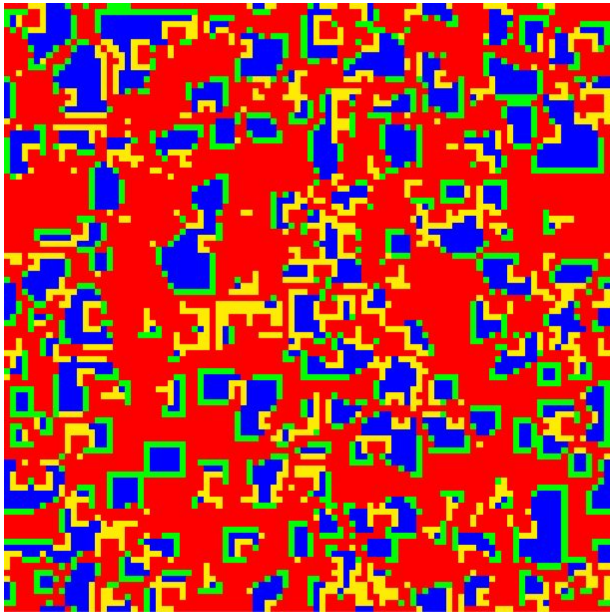
- *t=0* : 10% **D**, 90% **C**



- *b* = 1.74



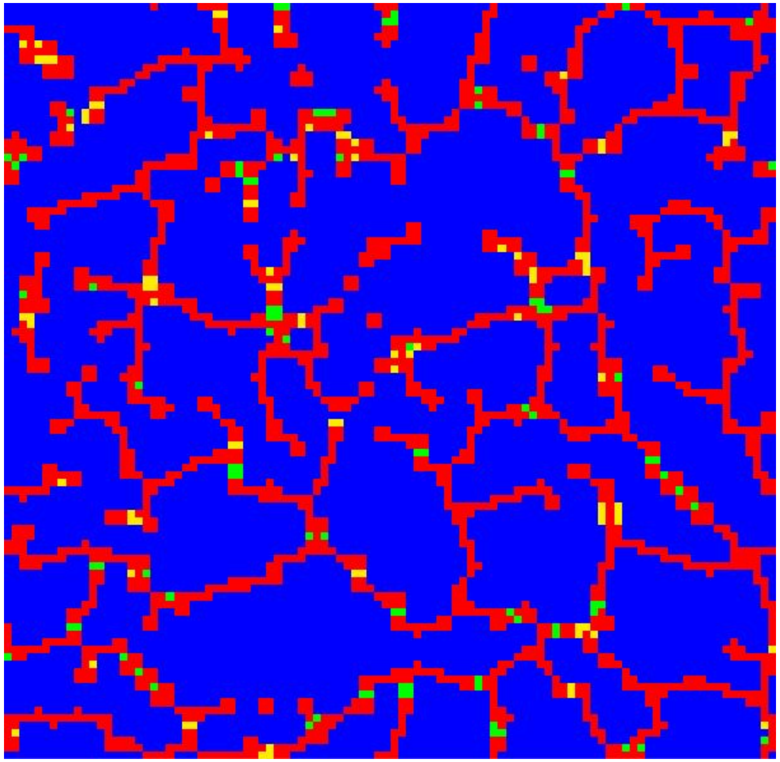
- *b* = 1.79



- *b* = 1.81

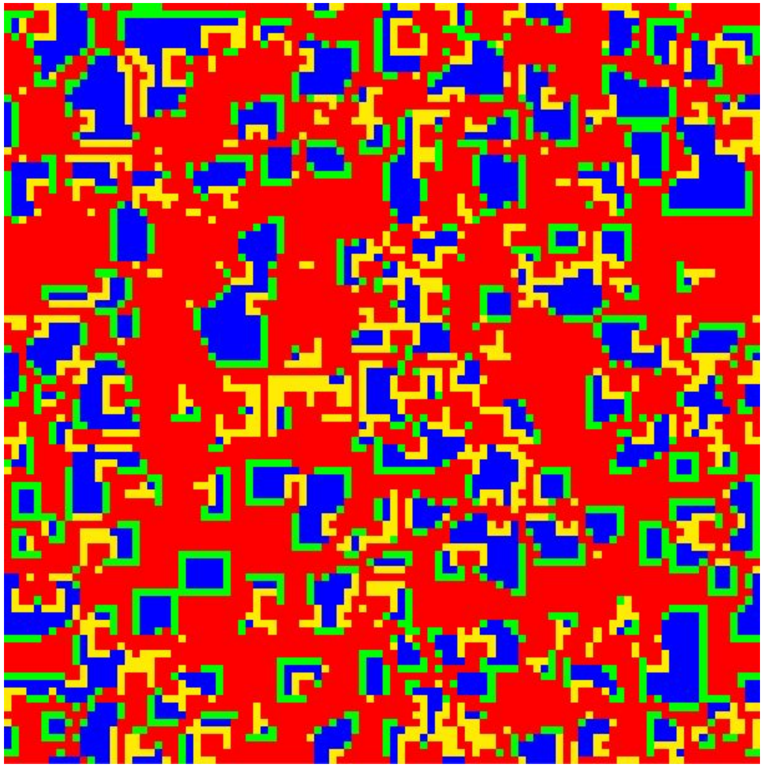
- Blue: **C**, red: **D**, yellow **C→D**, green: **D→C**.

b = 9/5 and around



- $b = 1.79$

- Nearly static web of **D.**



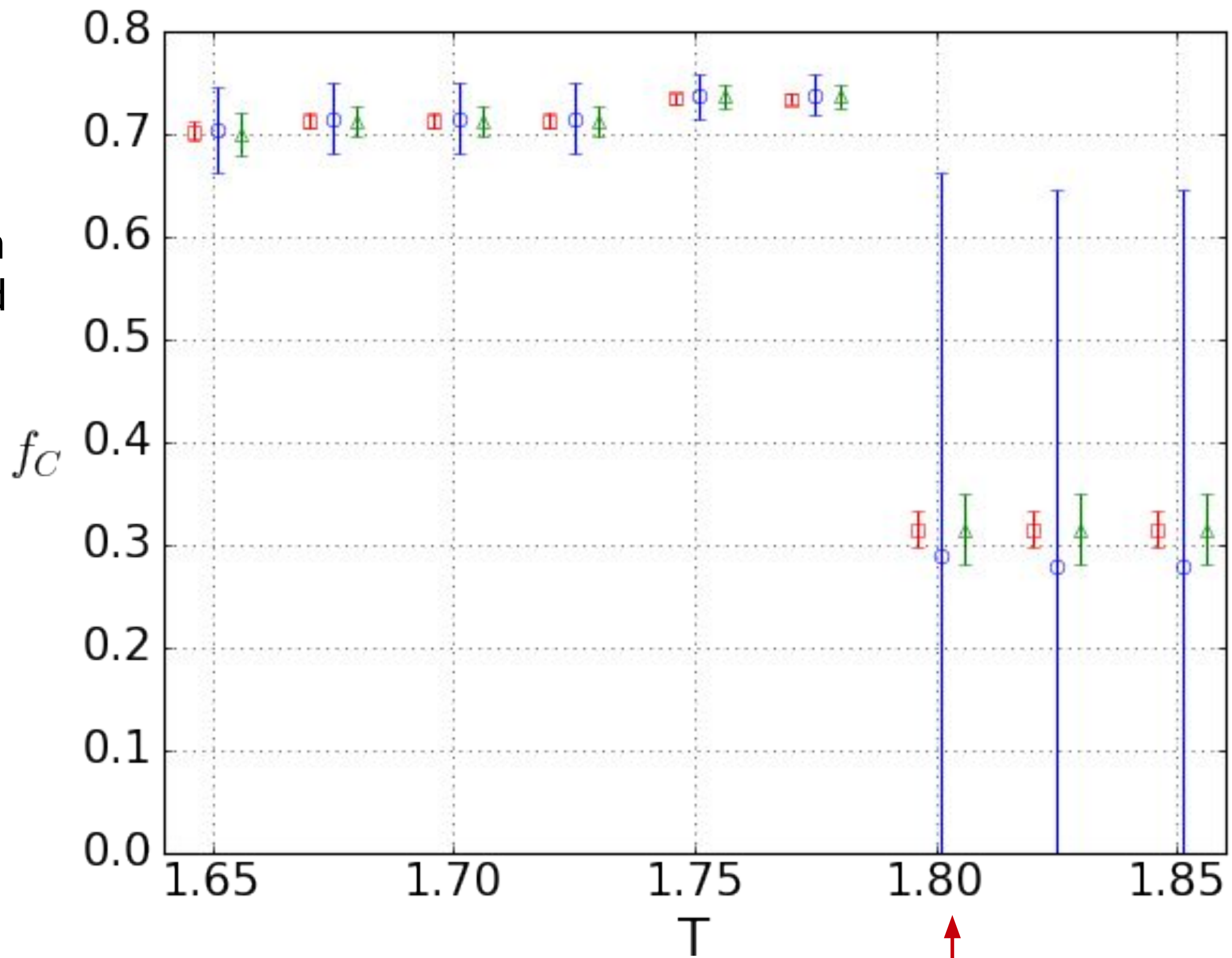
- $b = 1.81$

- Dynamic equlibrium at constant density.

Average density of cooperators

Color codes:

20 x 20: blue
50 x 50: green
100 x 100: red

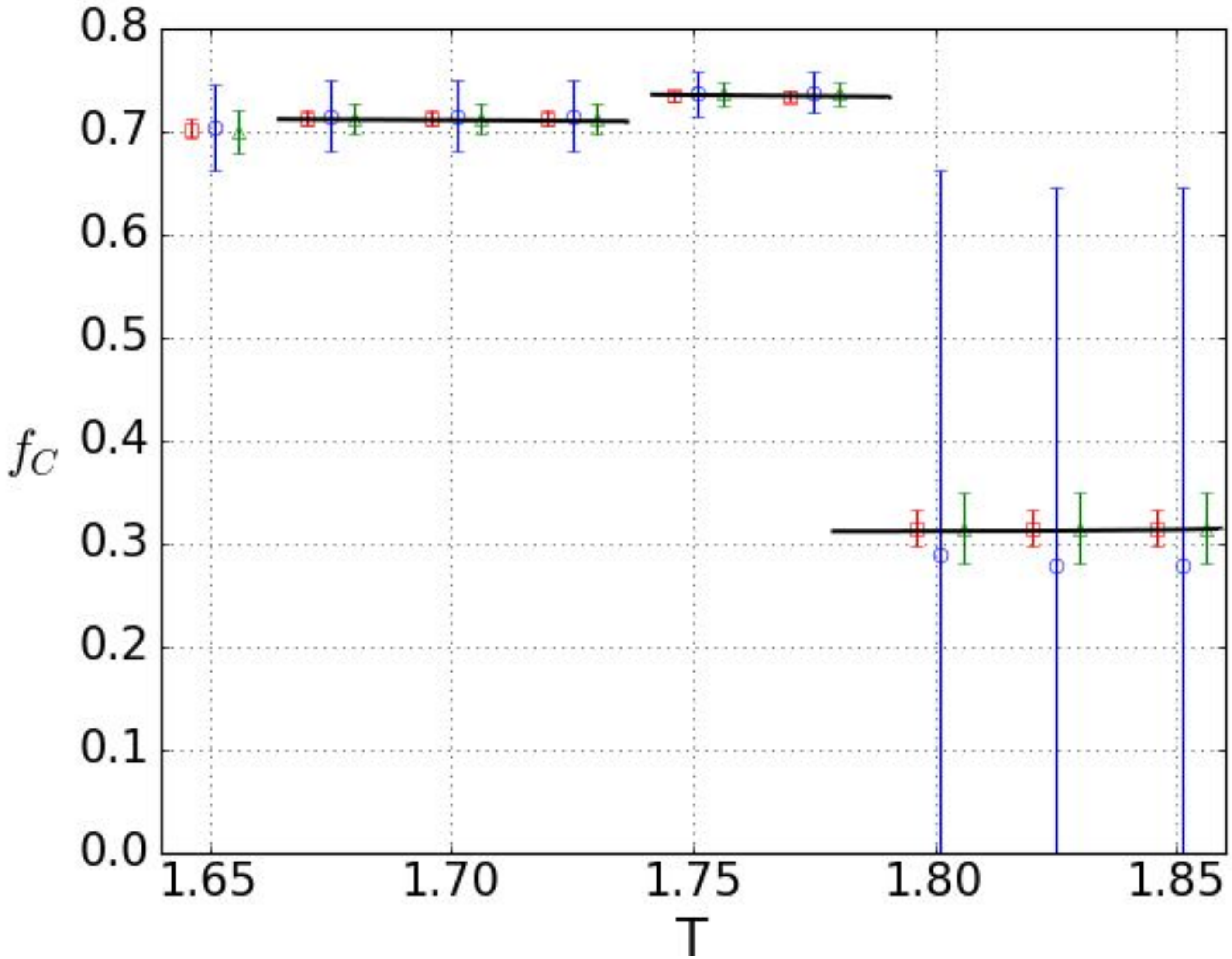


Something happens at b = 1.8

Average density of cooperators

Color codes:

20 x 20: blue
50 x 50: greer
100 x 100: rec

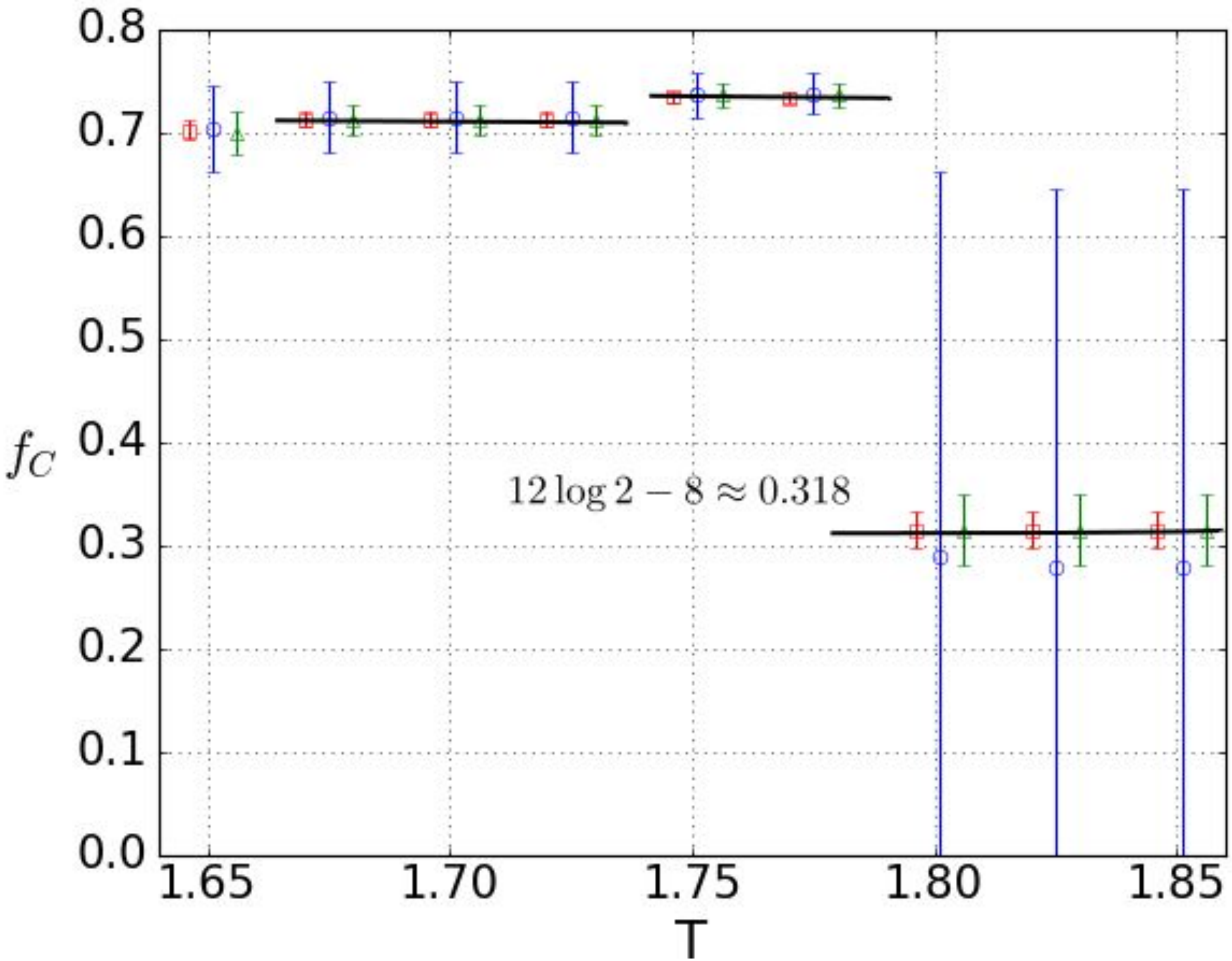


Density is constant in some ranges of *b*

Average density of cooperators

Color codes:

20 x 20: blue
50 x 50: green
100 x 100: red

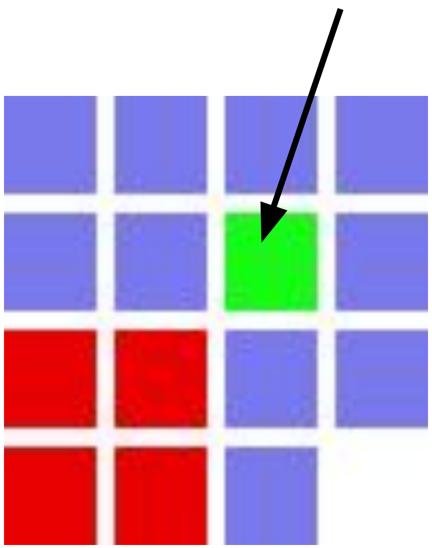


b > 1.8, density agrees with the value of Nowak & May, 1993

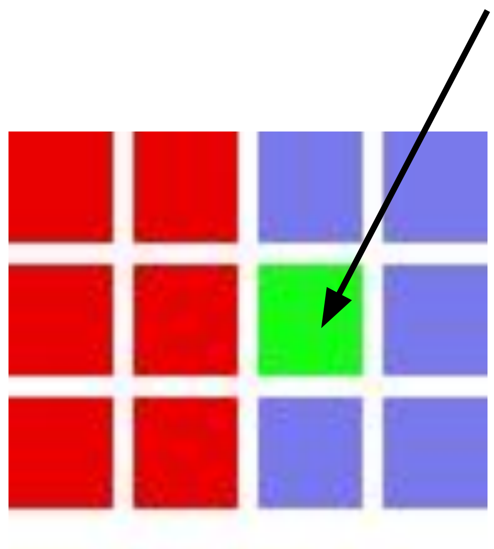
Evolution of local objects

A small cluster of defectors grows at the corners if *b > 1.8*

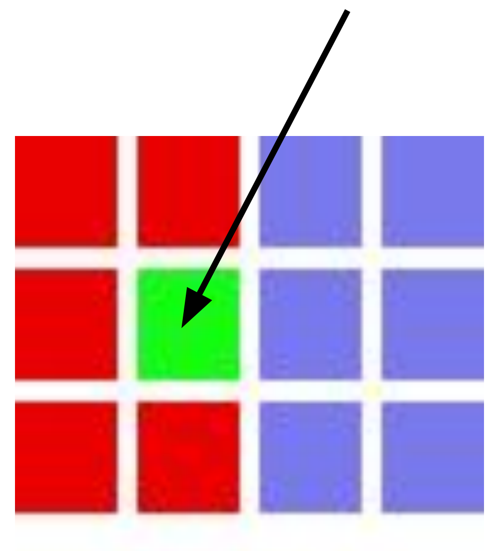
- **C→D** for *b* > 9/5 = 1.8



- **C→D** for *b* > 9/3 = 3

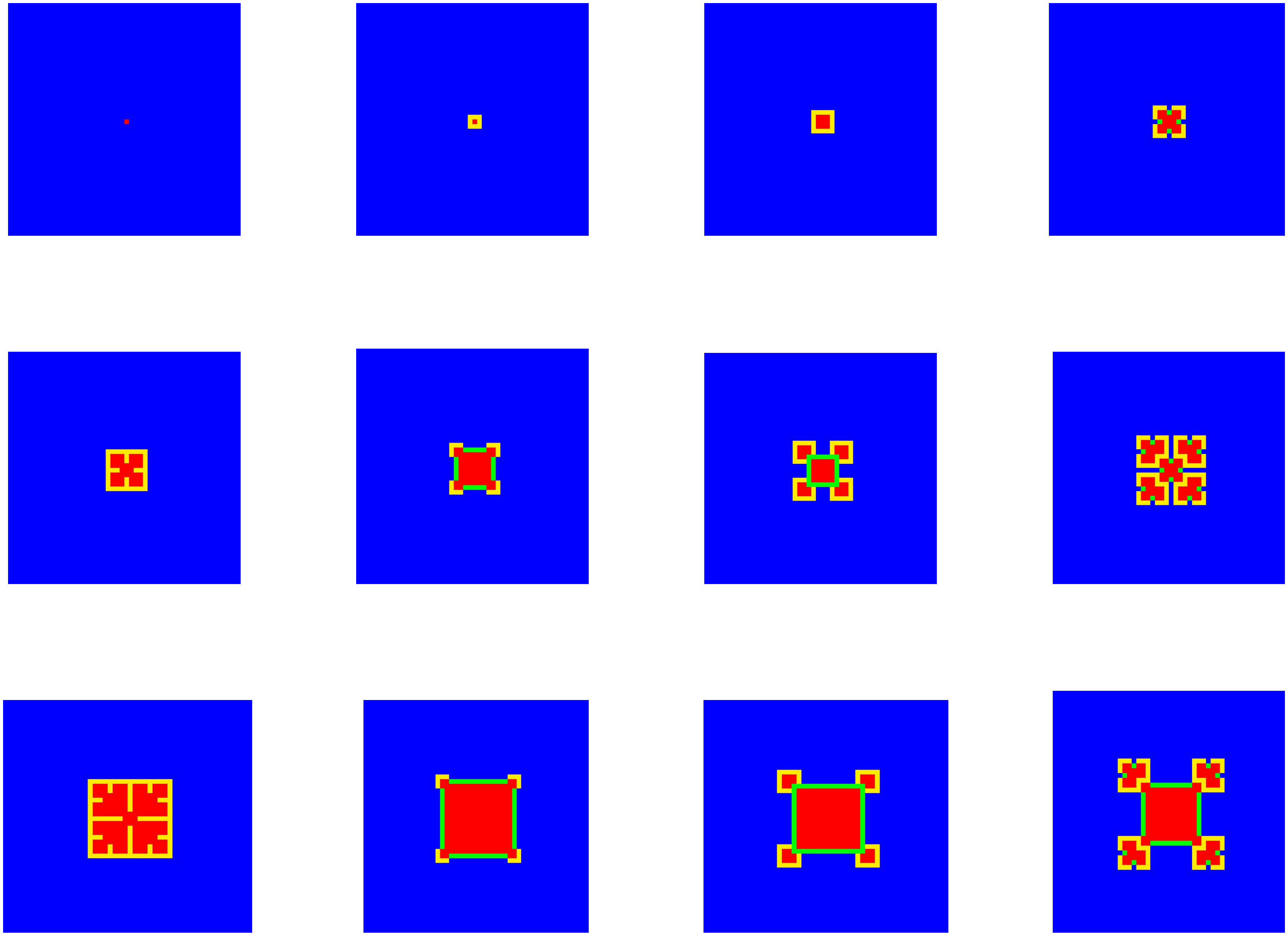


- **D→C** for *b* < 6/3 = 2



At finite density, clusters grow, shrink, fragment, collide etc

Evolution of local objects



Dynamic regimes and transitions

Discrete structure of the payoffs ⇒ distinct dynamic regimes, separated by transitions at special values of b (*Nowak and May, 1992*)

$$b = \ldots\ 8/5,\ 5/3,\ 7/4,\ 9/5,\ 2,\ 9/4,\ \ldots$$

The dynamics in a given regime is exactly the same for all b up to the transition points.

Нелокальная информация

Игра Новака-Мэя:

взаимодействие только с ближайшими соседями

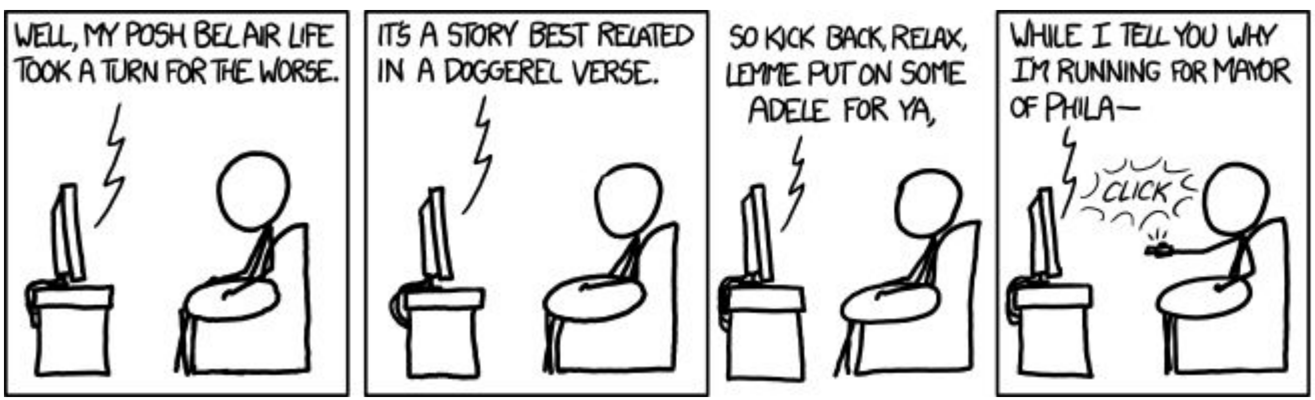
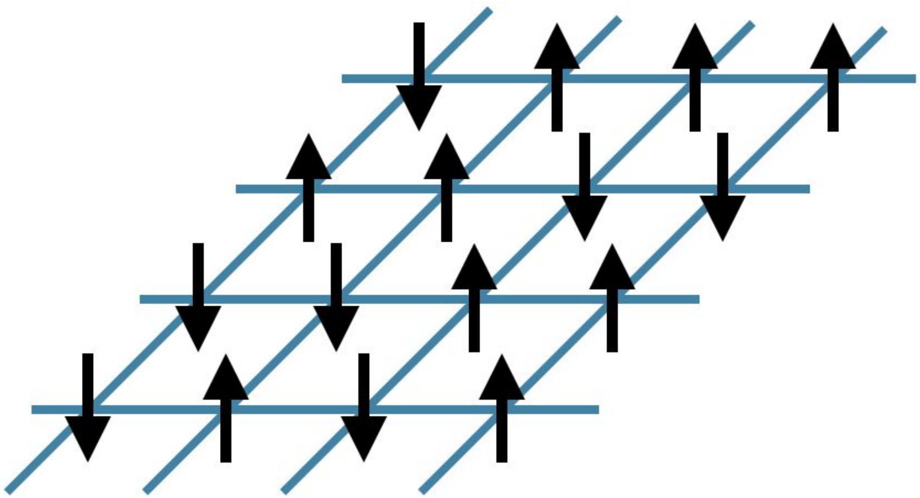
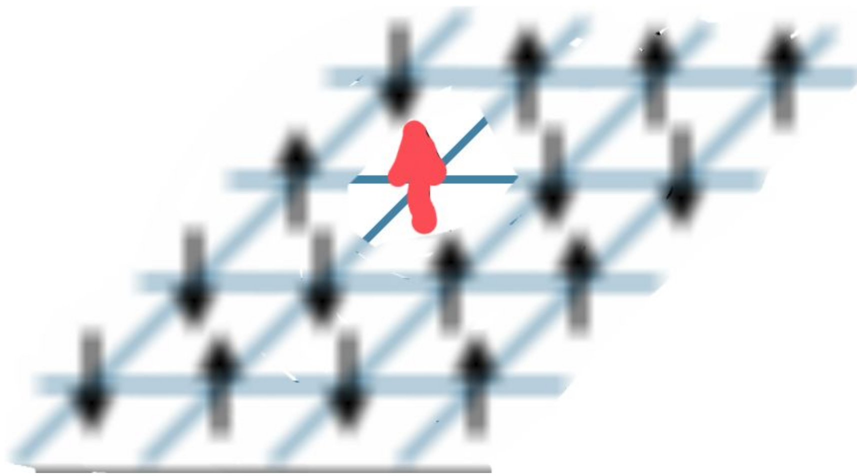


Image: Randall Monroe

Statistical physics POV: Mean field theory

P. Weiss, *L'hypothèse du champ moléculaire et la propriété ferromagnétique*, J. Phys Theor Appl **6**, 661 (1907)





Spin glasses, superconductivity, Bose-Einstein condensation, traffic flow etc etc etc

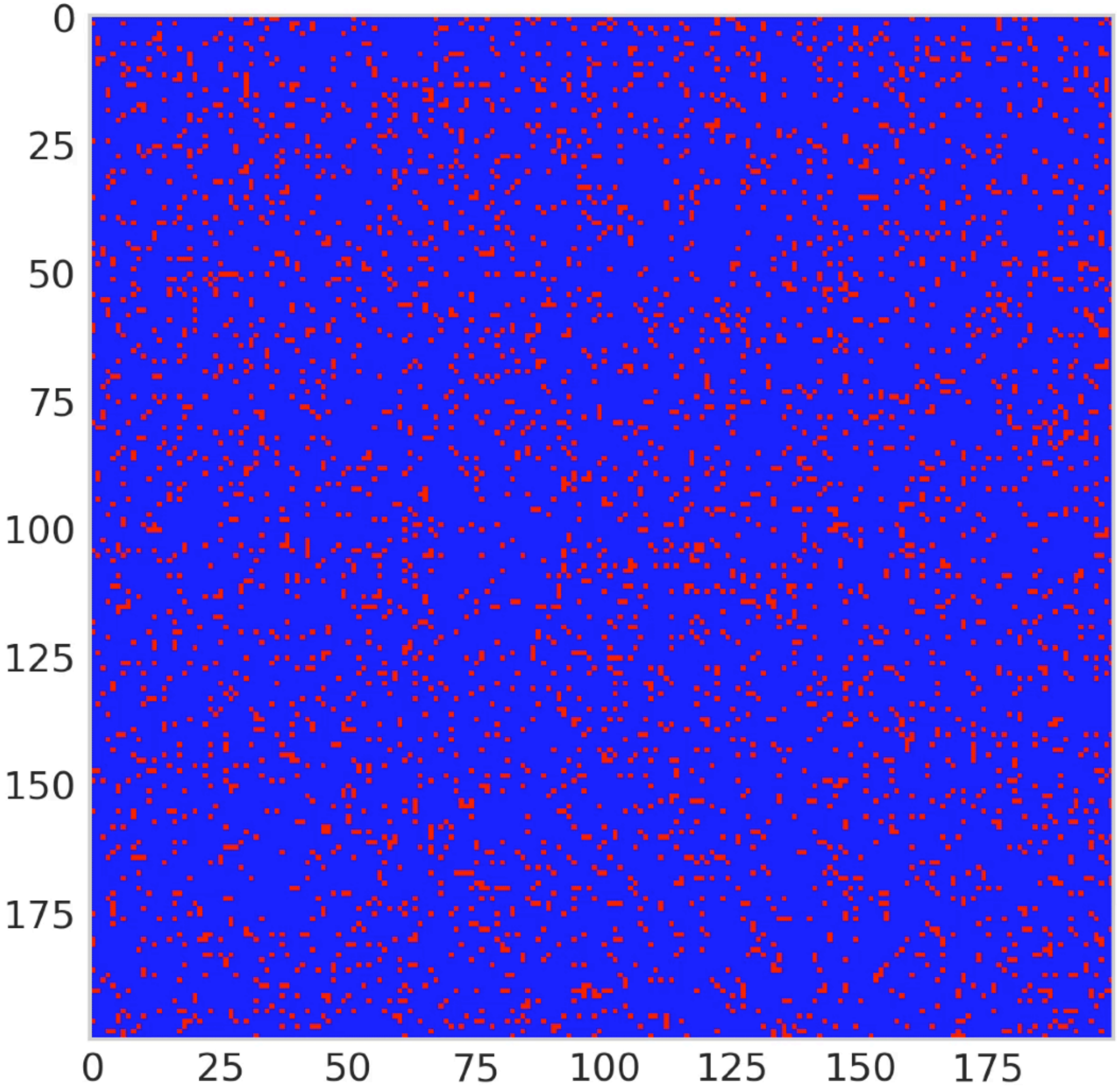
Prisoner's dilemma meets mean field

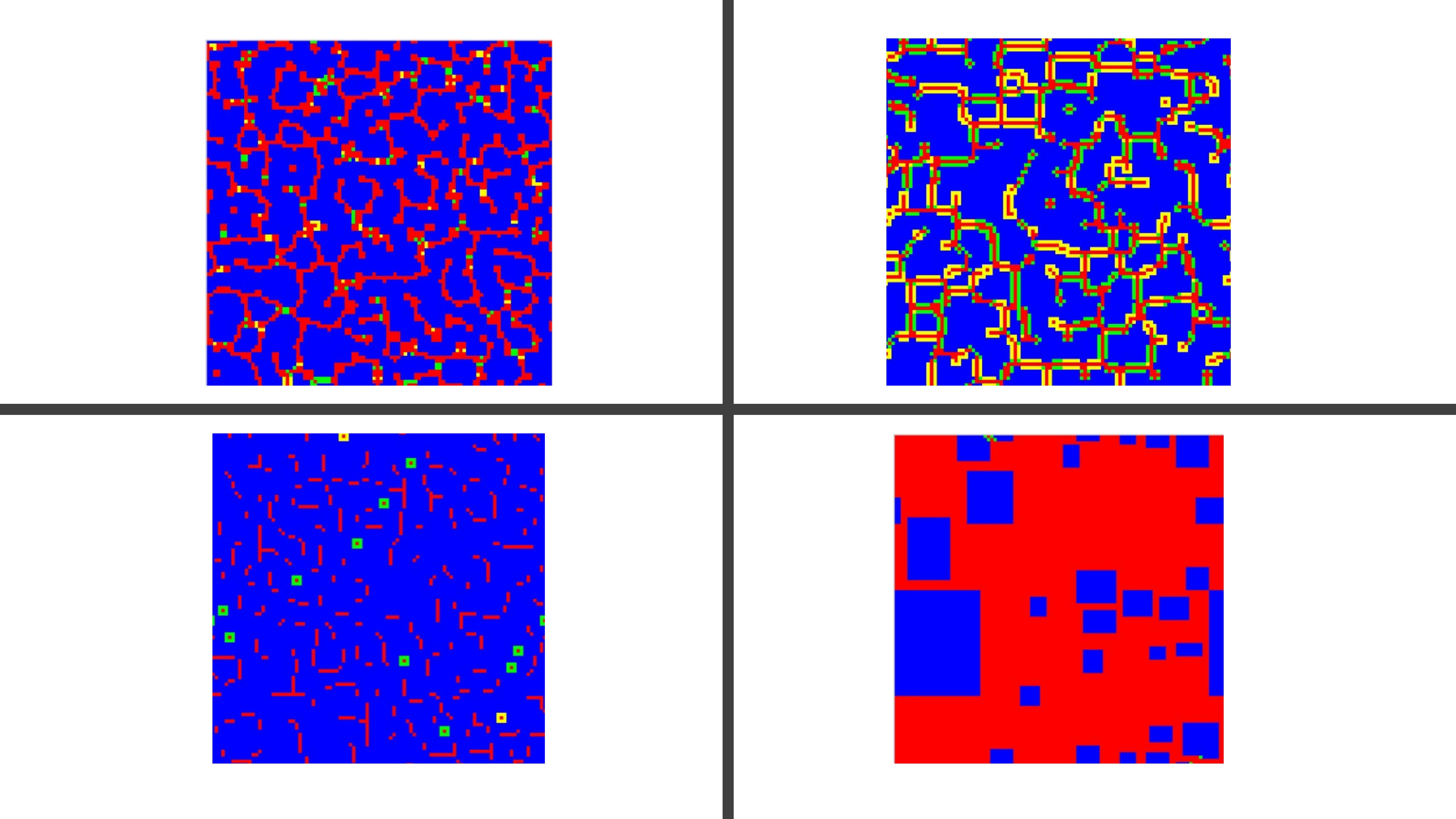
Агент

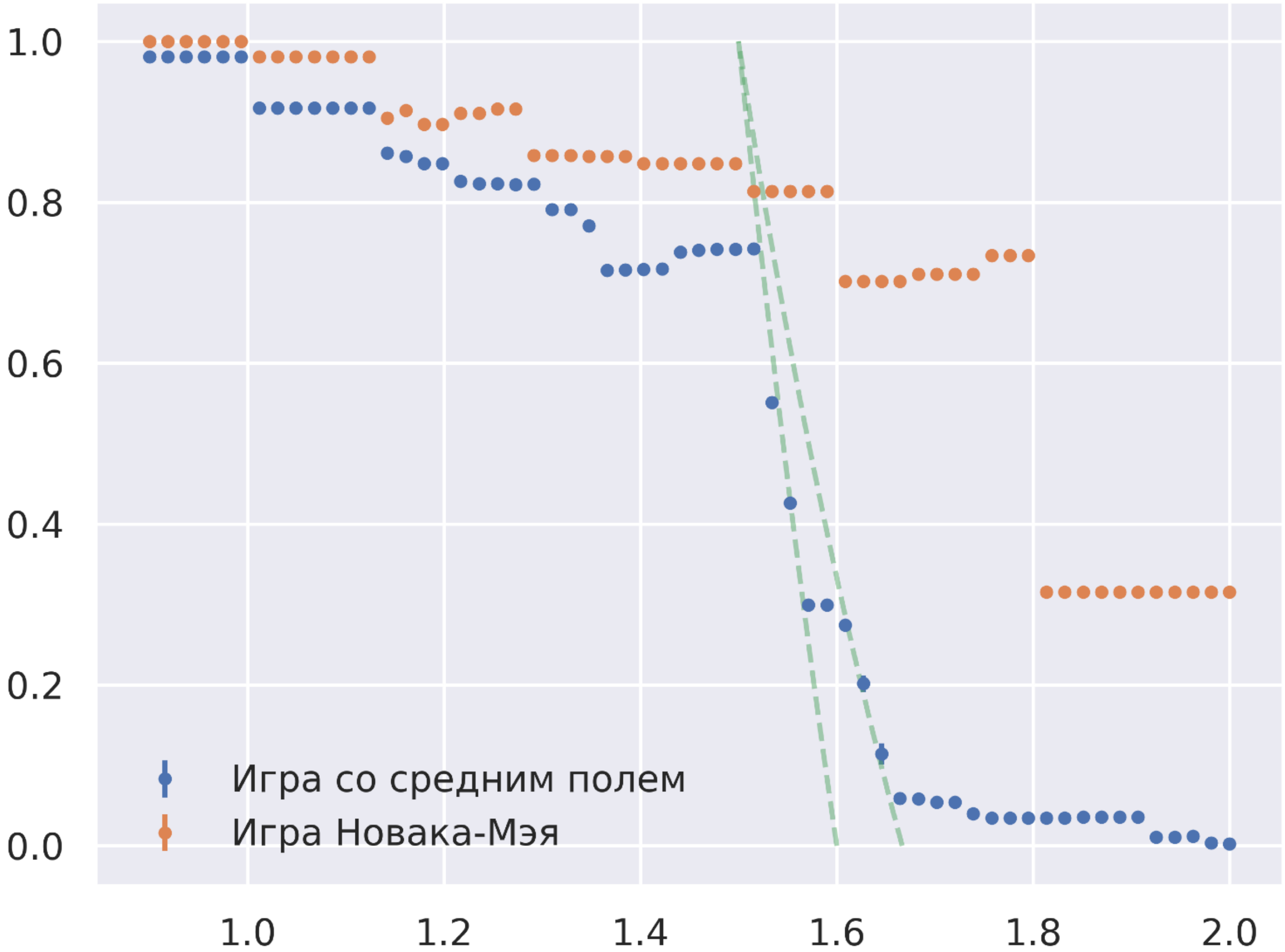
Сосед		$\mathcal{D}$	$\mathcal{C}$
	$\mathcal{D}$	0	0
	$\mathcal{C}$	b	1

Агент

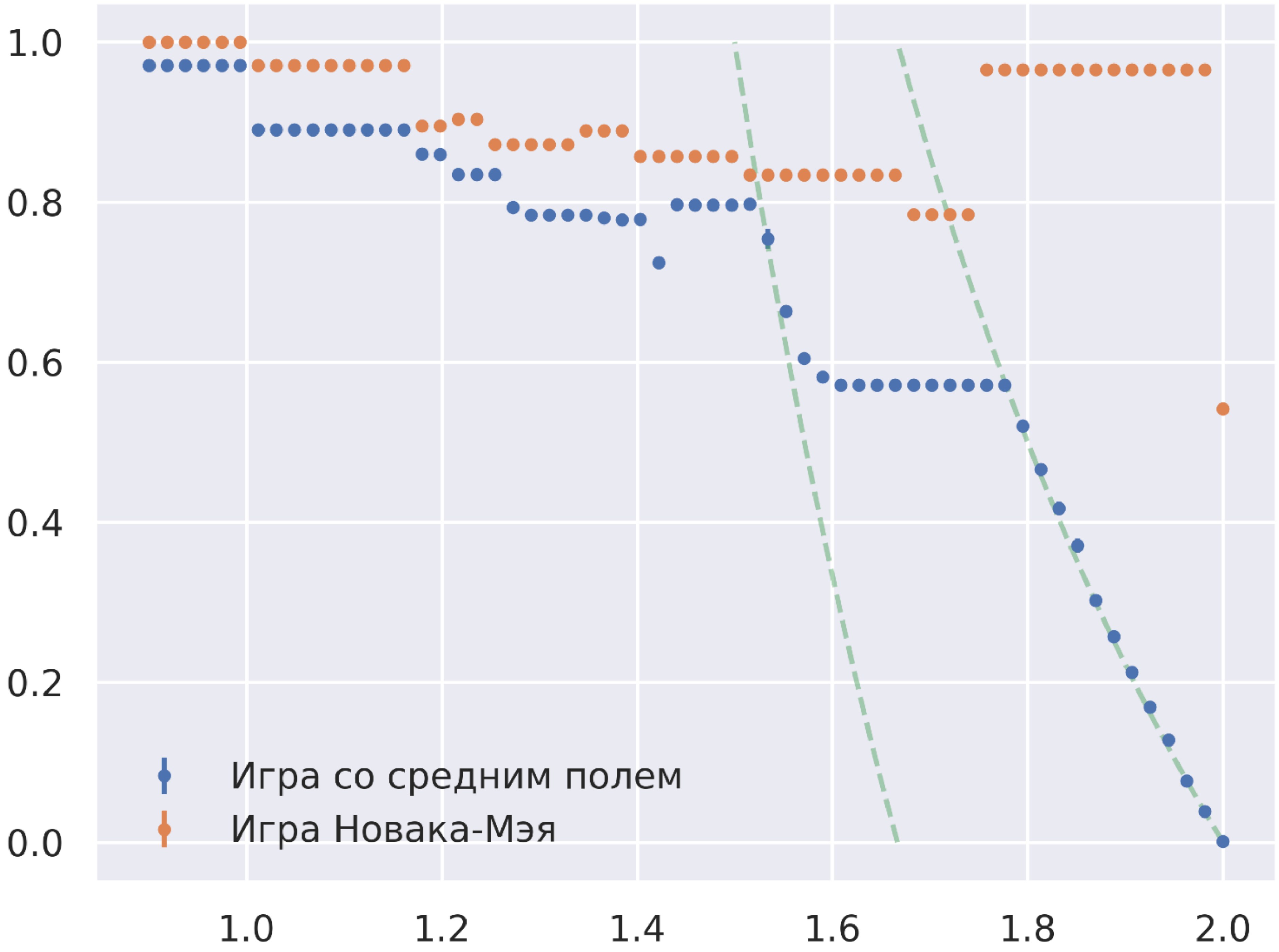
Сосед		$\mathcal{D}$	$\mathcal{C}$
	$\mathcal{D}$	0	0
	$\mathcal{C}$	bf_c	f_c


0
25
50
75
100
125
150
175
0
25
50
75
100
125
150
175





1.0
0.8
0.6
0.4
0.2
0.0
1.0
1.2
1.4
1.6
1.8
2.0
Игра со средним полем
Игра Новака-Мэя



1.0
0.8
0.6
0.4
0.2
0.0
1.0
1.2
1.4
1.6
1.8
2.0
Игра со средним полем
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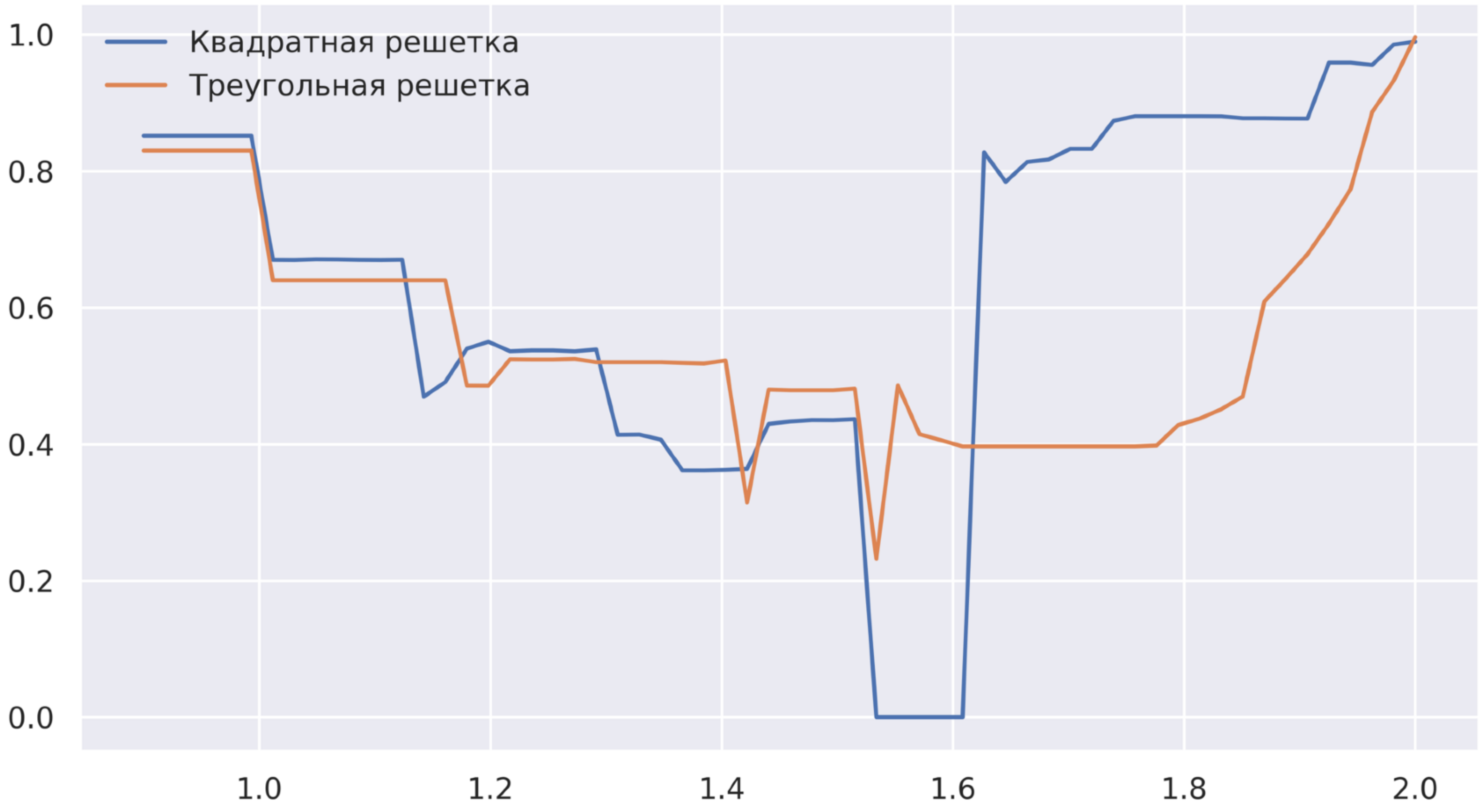
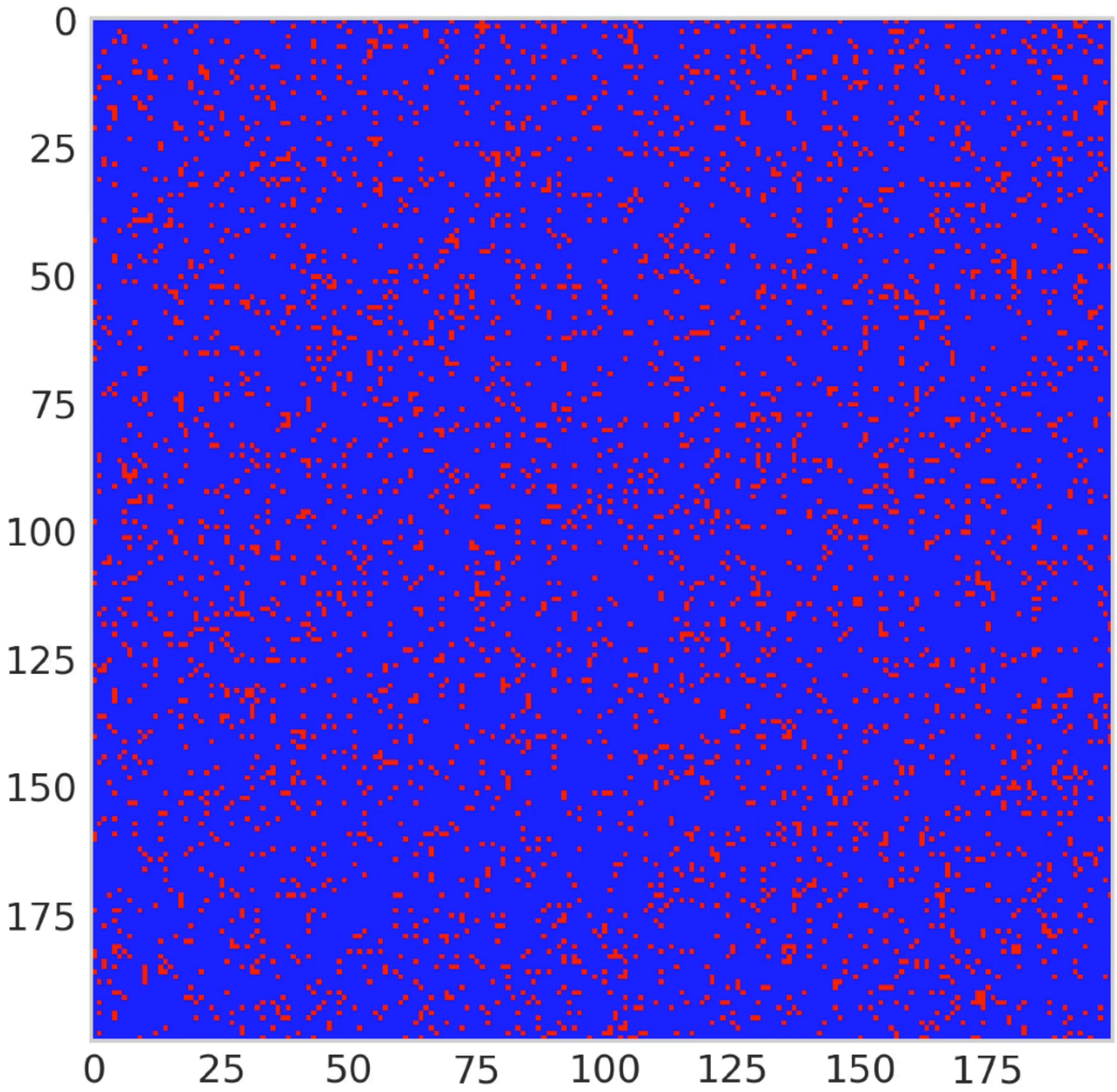


Квадратная решетка
Треугольная решетка
1.0
0.8
0.6
0.4
0.2
0.0
1.0
1.2
1.4
1.6
1.8
2.0


0
25
50
75
100
125
150
175
0
25
50
75
100
125
150
175

$$b = \frac{m + f_c}{n + f_c}$$

m, n = 1,...,8

$f_c \in [0, 1]$

Заключение

- Рассмотрена игра Новака-Мэя со средним полем
- Плотность может меняться непрерывно